

***T*-Parity Violation by Anomalies**

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Abstract

Little Higgs theories often rely on an internal parity (“ T -parity”) to suppress non-standard electroweak effects or to provide a dark matter candidate. We show that such a symmetry is generally broken by anomalies, as described by the Wess-Zumino-Witten term. We study a simple $SU(3) \times SU(3)/SU(3)$ Little Higgs scheme where we obtain a minimal form for the topological interactions of a single Higgs field. The results apply to more general models, including $[SU(3) \times SU(3)/SU(3)]^4$, $SU(5)/SO(5)$, and $SU(6)/Sp(6)$.

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INTRODUCTION

It is of urgent interest to discover the Higgs boson and determine whether it is (i) a fundamental particle, such as in SUSY theories or the Standard Model; (ii) a heavy, broad resonance, such as in TeV-scale dynamical models (like the top-seesaw [1]); or (iii) a light composite particle [2], such as in Little Higgs theories [3] with new dynamics at the ~ 10 TeV scale. Although (i) has received orders of magnitude more attention and is correspondingly more refined, we note that none of these options is presently ruled out by experiment. In option (iii) the Higgs boson is a (pseudo) Nambu-Goldstone boson (pNGB) of a spontaneously broken chiral symmetry. We focus presently on this possibility, as it is distinguished from (i) and (ii) by remarkable topological features. In the following we will not distinguish between “light composite Higgs boson” or “Little Higgs boson.”

In Little Higgs models some new UV dynamics at $\Lambda \sim 10$ TeV is imagined to produce a condensate, yielding pNGB mesons with decay constants of order $F \sim \Lambda/4\pi \sim 1$ TeV. Iso-doublet pNGB’s, analogues of the K -meson in QCD, play the role of Higgs scalar fields and develop VEV’s at the weak scale, $v/\sqrt{2} = 175$ GeV, breaking electroweak (EW) symmetry in the usual way. With suitable symmetries forbidding large mass corrections to the composite Higgs bosons, and a mechanism for generating the EW symmetry-breaking potential, one hopes to solve the “little hierarchy” problem with $\Lambda \sim 10$ TeV, $F \sim 1$ TeV and a naturally light Higgs boson at a scale of a few hundred GeV.

If the Higgs is a pNGB there will generally be “topological interactions” that reflect an anomaly structure of the underlying theory at scale Λ . These interactions, which are suppressed by powers of F , distinguish a chiral meson field from an ordinary field. This physics represents the holographic aspect of a chiral lagrangian theory. Topologically twisted field configurations of a $D = 5$ theory cast shadows on the $D = 4$ surface. These topological effects are contained in the Wess-Zumino-Witten (WZW) term [4–6], which must be included as part of the full effective action. The WZW term is a remarkable object and its full implications are well beyond the scope of the present paper. For a Little Higgs theory from a purely $D = 4$ perspective, it contains the full anomaly physics of the UV completion theory, expressed only in terms of gauge fields and pNGB’s [7–9]. It is specified by an integer quantity, *e.g.*, the number of “colors” of the constituent “techni-quarks”. The WZW interactions of Little Higgs bosons thus probe the underlying theory above the scale Λ , much

like the $\pi^0 \rightarrow \gamma\gamma$ interaction probes short-distance QCD.

Ignoring the WZW term would erroneously miss a significant part of the physics of any chiral lagrangian theory. For example, in many Little Higgs models it is natural to consider an apparent new symmetry dubbed “ T -parity” [10, 11]. Phenomenological studies suggest that incorporating such a symmetry can alleviate various fine tunings. The lightest T -odd particle has been suggested to act as a dark matter candidate.

We will show, however, that T -parity is generally violated by anomalies, and hence by the WZW term. This leads to the decay of the lightest T -odd particle into gauge fields, and other effects as well. Indeed, this would also happen in QCD where the π^0 can be viewed as a “ T -odd” field, under which $\pi^0 \rightarrow -\pi^0$, while the photon is “ T -even,” $A_\mu \rightarrow A_\mu$. T -parity is conserved in the QCD chiral Lagrangian without the WZW term, but is violated by anomalies. For example, the decay of the “dark matter candidate” π^0 proceeds via $\pi^0 \rightarrow \gamma\gamma$.

This is also the fate of T -parity in Little Higgs theories. Omitting the WZW term would lead to the *incorrect* conclusions that T -parity partners must be produced in pairs, and that the lightest T -odd state is stable. Including it leads to a rich set of new interactions that can probe the underlying UV completion of the effective theory at the fundamental scale Λ .

The WZW term could in principle be absent, but we know of no sensible UV theory in which that would be the case. The chiral lagrangian theory will unitarize itself into something new at scales above Λ , and a QCD-like theory in $D = 4$ is the most straightforward possibility. Alternatively, one might argue that the chiral lagrangian becomes a compactified $D \geq 5$ Yang-Mills theory. However, there is a rich set of topological objects in $D \geq 5$ that require Chern-Simons terms for their complete description. [25] Moreover, if the UV theory derives from extra dimension(s) we must still provide for the chirality of the ordinary quarks and leptons. This implies “chiral delocalization” in the extra dimensions, and in turn, Chern-Simons terms that holographically descend to the WZW term at low energies in $D = 4$.

We will study a simple composite Higgs model that imitates QCD, with the chiral symmetry breaking pattern $SU(3) \times SU(3) \rightarrow SU(3)$. The Higgs is identified with the kaon. The EW $SU(2) \times U(1)$ interactions are a subgroup of the vectorial $SU(3)$ subgroup and have no internal gauge anomalies. We then introduce a $U(1)_5$ axial vector gauge field, \tilde{B} , coupled to the λ^8 axial current. This situation imitates what happens in all Little Higgs theories with T -parity. We study the interactions of the Higgs, W , Z , γ and the T -odd field

\tilde{B} in the presence of the WZW term.

We will see that in a naive treatment there are Higgsless “Chern-Simons” operators generated by the WZW term, such as $\tilde{B}ZdZ$, and $\tilde{B}WdW$. [26] These operators carry gauge anomalies, and are a symptom of uncanceled \tilde{B} gauge anomalies in the UV completion theory. We briefly describe two methods of cancelling anomalies in the UV theory, a “lepton” sector, and a “mirror” sector. At the level of the WZW term it is easy to implement the effect of the anomaly cancellation sectors. When the anomalies are cancelled, the Chern-Simons operators drop out, as they must by gauge invariance. Gauge invariant operators involving the Higgs field remain, and these represent the universal T -parity violating topological interactions of the Higgs, W , Z , γ and \tilde{B} . As an example, we obtain the partial decay width of $\tilde{B} \rightarrow ZZ$ at order $\mathcal{O}(v^2/F^2)$. This clearly demonstrates that \tilde{B} cannot be a dark-matter candidate in this scheme.

We have thus derived the minimal form of a single Higgs fields participating in a T -parity violating interaction. The results can be applied to multi-Higgs boson extensions, or extensions with additional gauge bosons. We briefly discuss issues relevant to $[SU(3) \times SU(3)/SU(3)]^4$, $SU(5)/SO(5)$, and $SU(6)/Sp(6)$ models.

$SU(3) \times SU(3)/SU(3)$ LITTLE HIGGS MODEL

Consider a QCD-like theory with strong gauge group $SU(N_c)$, and with $SU(3)$ flavor triplets of techni-quarks, Ψ_L and Ψ_R , that transform in the fundamental representation with N_c colors. The strong interaction results in a condensate $\langle \psi_L^i \bar{\psi}_R^j \rangle \sim \Lambda^3 \delta^{ij}$, leading to an $SU(3)_L \times SU(3)_R \times U(1)/SU(3) \times U(1)$ chiral Lagrangian described by the 3×3 unitary matrix field $U^{ij} \sim \psi_L^i \bar{\psi}_R^j$: (we ignore the axial $U(1)$ pNGB, *i.e.*, the η')

$$U = \exp(2i\tilde{\pi}/F), \quad \tilde{\pi} = \sum_{a=1}^8 \pi^a \lambda^a / 2 = \frac{1}{2} \begin{pmatrix} \sum_{a=1}^3 \pi^a \tau^a + \eta \mathbb{1}_2 / \sqrt{3} & H \\ H^\dagger & -2\eta / \sqrt{3} \end{pmatrix}. \quad (1)$$

The Higgs doublet is identified with the kaon.

At the techni-quark level this system is coupled to left- and right-handed gauge fields, A_L and A_R respectively, which include the EW fields and additional new gauge interactions:

$$D_\mu \Psi_L = (\partial_\mu - iA_{L\mu})\Psi_L, \quad D_\mu \Psi_R = (\partial_\mu - iA_{R\mu})\Psi_R. \quad (2)$$

This induces the covariant derivative on U :

$$D_\mu U = \partial_\mu U - iA_{L\mu}U + iUA_{R\mu}. \quad (3)$$

or, in terms of vector fields, $V = \frac{1}{2}(A_R + A_L) = V^a T^a$, and axial vector fields, $V^5 = \frac{1}{2}(A_R - A_L) = V^{5a} T^a$:

$$D_\mu U = \partial_\mu U - i[V_\mu, U] + i\{V_\mu^5, U\}, \quad D_\mu \Psi = (\partial_\mu - iV_\mu - iV_\mu^5 \gamma_5) \Psi \quad (4)$$

The low-energy theory is specified in terms of U , V_μ and V_μ^5 . The axial vector fields eat mesons to acquire mass while the vector fields remain massless and transform in the adjoint under the diagonal $SU(3)$ subgroup.

T -Parity and “Reconstruction”

Following the terminology of Cheng and Low [10, 11] we can introduce the concept of “ T -parity”, under which the fields transform as $V \rightarrow +V$, $V^5 \rightarrow -V^5$ and $\tilde{\pi} \rightarrow -\tilde{\pi}$. This symmetry is independent of “space-parity”, under which $(x^0, \vec{x}) \rightarrow (x^0, -\vec{x})$, $\tilde{\pi} \rightarrow \tilde{\pi}$, $(V_0, \vec{V}) \rightarrow (V_0, -\vec{V})$, and $(V_0^5, \vec{V}^5) \rightarrow (V_0^5, -\vec{V}^5)$. Both T -parity and space-parity are symmetries of the “ordinary” chiral Lagrangian of QCD,

$$\mathcal{L} = -\frac{1}{2}\text{Tr}(F_{\mu\nu}^2) + \frac{1}{2}F^2\text{Tr}(D_\mu U D_\mu U^\dagger + \dots), \quad (5)$$

where the ellipsis denotes other invariant combinations of U and D_μ . It is not possible to write a manifestly local (*i.e.*, four-dimensional), and globally chiral-invariant term that breaks T -parity, and thus the symmetry would appear to be exact [27].

Any $D = 4$ chiral lagrangian theory can be viewed as a deconstructed gauge theory in $D = 5$ [13]. We can “reconstruct” the $D = 4$ chiral lagrangian of QCD, mapping it into a corresponding $D = 5$ Yang-Mills theory of flavor. “ T -parity” is then seen to be equivalent to “KK-mode parity”. We consider a manifold in $D = 5$ with boundary branes at $y = 0$ and $y = R$. On the manifold we have a bulk $SU(3)$ (flavor) Yang-Mills gauge field $B_A = B_A^a \lambda^a / 2$. If we apply the boundary condition $[D_5, F_{\mu\nu}] = 0$ to the gauge fields on the branes we obtain the spectrum of QCD. The general configurations of B_A that are allowed on the y interval are classified in terms of reflections about $y = R/2$. A B_μ zero mode, constant in y , is even under this reflection and is identified with V_μ , and assigned T -parity +1 (this corresponds

to photons, or other fundamental vector gauge fields, or to the ρ octet). The B_5 zero-mode, identified with $\tilde{\pi}$, is constant and thus also even under reflection about $y = R/2$, but is the 5th component of a 5-vector and thus assigned T -parity -1 . The first KK-mode of B_μ is identified with V_μ^5 . It has wave-function $\cos(\pi y/R)$ (the mode $\sin(\pi y/R)$ is forbidden by the b.c.'s) and is odd under reflection about $y = R/2$, and thus assigned T -parity -1 . This mode eats the first $\sin(\pi y/R)$ KK mode of B_5 which is also T -parity even. The first physical KK-mode of B_5 (corresponding to the a^0 octet) has wave-function $\cos(\pi y/R)$ and is thus assigned T -parity of $+1$, and so forth.

T -parity is, however, not a good symmetry of QCD. For example, $\pi^0 \rightarrow \gamma\gamma$ is an allowed process, and we also have that the ϕ meson (the V^8 in our notation) decays both to $K\bar{K}$ and to $\pi\pi\pi$. There is no assignment of a conserved T -parity that is consistent with these facts. The resolution is that we must incorporate the WZW term into the effective action. The WZW term is four-dimensional and globally chiral invariant, although not manifestly so. For general gauge fields, it is not gauge invariant, reflecting the anomaly structure of the underlying QCD theory. However, when only non-anomalous (*e.g.*, vector) generators are gauged, it is gauge invariant, albeit again not manifestly so. The WZW term is odd under independent space-parity or T -parity reflections, and only the combination of these two parities survives as the true parity symmetry.

From the perspective of the $D = 5$ reconstructed theory, the WZW term is seen to arise from the Chern-Simons term which involves $\epsilon^{ABCDE} B_A \partial_B B_C \partial_C B_D + \dots$. Under compactification the Chern-Simons term resolves into two terms, $\sim B_5 \epsilon^{\mu\nu\rho\sigma} \partial_\mu B_\nu \partial_\rho B_\sigma$ and $\sim \epsilon^{\mu\nu\rho\sigma} B_\mu \partial_5 B_\nu \partial_\rho B_\sigma$. Both terms break “KK-mode parity” while conserving overall $D = 5$ parity. Again, the low-energy effective action contains only a single overall parity symmetry. We remark that *any higher dimension theory*, *e.g.*, Randall-Sundrum models, with B_5 as a Higgs field and with chiral fermion delocalization, will have CS term effects. Moreover, a theory with Ψ_L (Ψ_R) fermions on the $y = 0$ ($y = R$) brane, requires the Chern-Simons term for anomaly cancellation. The full WZW term arises from the Chern-Simons term and the Dirac determinant when we integrate out the fermions.

Let us focus on the gauge/Higgs sector of the model, ignoring the ordinary standard model quarks and leptons, and the mechanism that gives rise to the Higgs potential. Consider gauging the $SU(2) \times U(1)$ subgroup of the vectorial $SU(3)$, and a $U(1)_5$ axial subgroup of

$SU(3) \times SU(3)$. We specify this by the covariant derivative of Eq.(4) and the gauge fields:

$$V_\mu = \sum_{a=1}^3 g_2 W_\mu^a \lambda^a / 2 - g_1 B_\mu \lambda^8 / 2\sqrt{3} \quad V_\mu^5 = \tilde{g} \tilde{B}_\mu \lambda^8 / 2. \quad (6)$$

The axial vector field \tilde{B} eats the η and becomes massive, with a mass:

$$m_{\tilde{B}} = \tilde{g} F \quad (7)$$

The Higgs field transforms as an isodoublet, with covariant derivative

$$D_\mu H = \partial_\mu H - i g_2 W_\mu^a \frac{\tau^a}{2} H + \frac{i}{2} g_1 B_\mu H \quad (8)$$

The T -parity conserving couplings of \tilde{B} in unitary gauge (in which it eats the η) can be inferred from the full Eq.(5).

This gauging is anomaly-free with respect to the color gauge group, but introduces $SU(2)^2 \times U(1)_5$ and various $U(1)_5 U(1)^2$ and $U(1)_5^3$ anomalies. These anomalies can be cancelled either by a spectator “lepton” sector, or by introducing a “mirror sector” with the opposite chirality, as we discuss in the following section.

The WZW action can be evaluated straightforwardly as in QCD. We use the form of Kaymakcalan, Rajeev and Schechter [14] (their Eq.(4.18)). Relevant issues for adapting this to Little Higgs theories are described in our earlier paper [5]. We work in unitary gauge for the heavy fields, where \tilde{B} eats η . Through order $1/F^2$, we find:

$$\begin{aligned} \Gamma_{WZW} = \int d^4x \frac{\tilde{g} N_c}{24\sqrt{3}\pi^2} \epsilon^{\mu\nu\rho\sigma} \tilde{B}_\mu [& \\ & -\frac{1}{3} g_1^2 [B_\nu \partial_\rho B_\sigma] + 2g_2^2 \text{Tr}[W_\nu \partial_\rho W_\sigma] - \frac{3ig_2^3}{2} \text{Tr}[W_\nu W_\rho W_\sigma] \\ & -\frac{ig_1}{4F^2} F_{\nu\rho}^B [H^\dagger (D_\sigma H) - (D_\sigma H^\dagger) H] - \frac{ig_2}{F^2} [H^\dagger F_{\nu\rho}^W (D_\sigma H) - (D_\nu H^\dagger) F_{\rho\sigma}^W H]] , \end{aligned} \quad (9)$$

where DH is given in Eq.(8). Here $F_{\mu\nu}^W$ and $F_{\mu\nu}^B$ are field strengths for the B , $F_{\mu\nu}^B = 2\partial_{[\mu} B_{\nu]}$, and W : [Square brackets around indices denote antisymmetrization, $A_{[\mu} B_{\nu]} \equiv \frac{1}{2}(A_\mu B_\nu - B_\mu A_\nu)$]

$$F_{\mu\nu}^W = \begin{pmatrix} \partial_{[\mu} W_{\nu]}^3 - ig_2 W_{[\mu}^+ W_{\nu]}^- & \sqrt{2}(\partial_{[\mu} W_{\nu]}^+ - ig_2 W_{[\mu}^3 W_{\nu]}^+) \\ \sqrt{2}(\partial_{[\mu} W_{\nu]}^- + ig_2 W_{[\mu}^3 W_{\nu]}^-) & -\partial_{[\mu} W_{\nu]}^3 + ig_2 W_{[\mu}^+ W_{\nu]}^- \end{pmatrix} \quad (10)$$

where

$$W_\mu^3 = Z_\mu^0 \cos \theta_W + A_\mu \sin \theta_W \quad B_\mu = -Z_\mu^0 \sin \theta_W + A_\mu \cos \theta_W. \quad (11)$$

Here A_μ (Z_μ^0) is the physical photon (Z^0) vector potential.

However, any action containing Eq.(9) alone, cannot be physically correct. The terms of Eq.(9) containing $\tilde{B}W\partial W$ and $\tilde{B}B\partial B$ generate anomalies and describe the disallowed decay of a massive spin-1 field into two massless spin-1 fields in violation of gauge invariance and the Landau-Yang theorem (the ρ cannot decay to two photons!). This is a symptom of the fact that the axial λ^8 symmetry of \tilde{B} is anomalous at the level of our fundamental techni-quark theory in Eq.(2). A consistent theory requires additional anomaly-cancelling structure, and this structure will modify the decay amplitudes. We will see that it is easy to represent this in a fairly general way.

Anomaly cancellation

A “lepton sector” can be constructed that cancels gauge anomalies and makes the model consistent. Consider, *e.g.*, N_c “leptons” with covariant derivative:

$$D_\mu L = \partial_\mu L - iV_\mu L + iV_\mu^5 \gamma^5 L. \quad (12)$$

Here we have flipped the sign of the V_μ^5 interaction for the leptons relative to the techni-quarks in Eq.(4). We postulate that the leptons acquire mass via their own Higgs mechanism, with the gauge invariant Lagrangian:

$$\Delta\mathcal{L} = \frac{1}{2}f^2\text{Tr}(D_\mu U' D_\mu U'^\dagger) + \bar{L}i\not{D}L - m_L(\bar{L}_L U' L_R + h.c.) , \quad (13)$$

and here we must introduce an “axion” field, a to preserve the axial λ^8 symmetry:

$$U' = e^{2ia\lambda^8/f} . \quad (14)$$

Under $U(1)_5$ we have $L_R \rightarrow e^{i\theta\lambda^8} L_R$, $L_L \rightarrow e^{-i\theta\lambda^8} L_L$ and the axion and \tilde{B} transform as:

$$\tilde{g}\tilde{B}_\mu \rightarrow \tilde{g}\tilde{B}_\mu + \partial_\mu\theta, \quad a/f \rightarrow a/f - \theta. \quad (15)$$

With the leptons and techni-quarks the theory is now anomaly free. Integrating out the leptons in a large mass limit will give a new WZW action,

$$\Gamma_{WZW} = \Gamma_{WZW}(U, \tilde{B}, W, B) - \Gamma_{WZW}(U', \tilde{B}, W, B). \quad (16)$$

The relative minus sign comes from flipping the \tilde{B} coupling constant sign in Eq.(12).

Note that integrating out the leptons is a convenience in that they would otherwise complicate the computation of physical processes. If the leptons were light we would still have overall anomaly cancellation, but we would have to compute open lepton processes and add them to WZW processes of the techni-quarks.

With anomaly cancellation in place we see immediately that the offending Chern-Simons terms, $\tilde{B}W\partial W$ and $\tilde{B}B\partial B$, cancel in the sum of WZW terms. There remain terms that involve the Higgs fields of the form $\sim H^\dagger H \tilde{B}W\partial W$ and $\sim H^\dagger H \tilde{B}B\partial B$. The scalar axion field a remains in the low-energy spectrum, and is neutral under EW $SU(2) \times U(1)$. Since we are not interested in the fate of the axion and the η , or the π pNGB's, presently we can simply set them to zero and generate the physical amplitudes of interest using the formula:

$$\Gamma_{WZW} \approx \Gamma_{WZW}(U, \tilde{B}, W, B) - \Gamma_{WZW}(1, \tilde{B}, W, B). \quad (17)$$

An alternative scheme that doesn't involve leptons or a fundamental axionic pseudoscalar field can be constructed by supposing the mirror fermions are themselves coupled to a strong gauge force, causing a mirror fermion condensate. Gauging both sectors in an identical manner,

$$\begin{aligned} U_1 &\sim \Psi_{1L} \bar{\Psi}_{1R} \rightarrow e^{i\epsilon_L} U_1 e^{-i\epsilon_R}, \\ U_2 &\sim \Psi_{2R} \bar{\Psi}_{2L} \rightarrow e^{i\epsilon_L} U_2 e^{-i\epsilon_R}, \end{aligned} \quad (18)$$

again ensures that all Chern-Simons terms and gauge anomalies cancel between the two sectors. The low-energy theory becomes a two-Higgs doublet model, with the anomaly physics described by $\Gamma_{WZW}(U_1) - \Gamma_{WZW}(U_2)$.

THE PHYSICAL T-PARITY VIOLATING WZW INTERACTION

Including the minimal anomaly cancelling spectators discussed above, and neglecting additional pNGB's, we see that the Chern-Simons terms cancel and we are left with the physical WZW term interaction of a single Little Higgs doublet with \tilde{B} through order $1/F^2$:

$$\begin{aligned} \Gamma_{WZW} = & \int d^4x \frac{\tilde{g} N_c}{24\sqrt{3}\pi^2} \epsilon^{\mu\nu\rho\sigma} \tilde{B}_\mu [\\ & -\frac{ig_1}{4F^2} F_{\nu\rho}^B [H^\dagger (D_\sigma H) - (D_\sigma H^\dagger) H] - \frac{ig_2}{F^2} [H^\dagger F_{\nu\rho}^W (D_\sigma H) - (D_\nu H^\dagger) F_{\rho\sigma}^W H]] . \end{aligned} \quad (19)$$

Note the pair of operators that are generated at this level of the form $\epsilon^{\mu\nu\rho\sigma} \tilde{B}_\mu [H^\dagger F_{\nu\rho} (D_\sigma H) - (D_\nu H^\dagger) F_{\rho\sigma} H]$. This is a generic operator structure for gauge/Higgs interactions in the WZW term for all Little Higgs theories containing a tree-level T -parity.

Since the theory is now anomaly free we can pass to unitary gauge whence the Higgs field takes the form:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h^0 \\ 0 \end{pmatrix}, \quad (20)$$

with $v = 246$ GeV. The interaction takes the explicit form:

$$\begin{aligned} \Gamma_{WZW} = & \frac{-\tilde{g}g_2^2 N_c}{96\sqrt{3}\pi^2 F^2} \int d^4x (v + h^0)^2 \epsilon^{\mu\nu\rho\sigma} \tilde{B}_\mu \times \\ & \left[2\sqrt{1 + \tan^2 \theta} (\partial_\nu Z_\rho^0 \cos \theta + \partial_\nu A_\rho \sin \theta - ig_2 W_\nu^+ W_\rho^-) Z_\sigma^0 \right. \\ & + 2 [(D_\nu^A W_\rho^+) W_\sigma^- + (D_\nu^A W_\rho^-) W_\sigma^+] - 4ig_2 \cos \theta Z_\nu^0 W_\rho^+ W_\sigma^- \\ & \left. - \tan \theta \sqrt{1 + \tan^2 \theta} (\partial_\nu Z_\rho^0 \sin \theta - \partial_\nu A_\rho \cos \theta) Z_\sigma^0 \right] \end{aligned} \quad (21)$$

where $D_\nu^A W_\nu^\pm = (\partial_\nu \mp ieA_\mu)W_\nu^\pm$ and A_μ is the photon vector potential. Note that Eq.(21), which is written in unitary gauge for the \tilde{B} , W , and Z , is manifestly invariant under electromagnetic gauge transformations.

Brief Survey of Physical Processes

Physical processes described by Eq.(21) all violate T -parity (and space-parity), conserving overall parity. Some examples worthy of study for future colliders include:

$$\begin{aligned} e^+ e^- \text{ or } q\bar{q} \text{ or } \mu^+ \mu^- & \rightarrow (\gamma^*, Z^*) \rightarrow \tilde{B} + Z; \tilde{B} + \gamma; \tilde{B} + WW \\ e^+ e^- \text{ or } q\bar{q} \text{ or } \mu^+ \mu^- & \rightarrow (\gamma^*, Z^*) \rightarrow \tilde{B} + h^0; \tilde{B} + 2h^0 \\ q\bar{q} & \rightarrow W^* \rightarrow \tilde{B} + W; \tilde{B} + W + h^0; \tilde{B} + W + 2h^0 \end{aligned} \quad (22)$$

These processes could in principle be used to measure N_c for the UV completion theory. They are, however, suppressed in rate by N_c^2/F^4 and would probably be best suited for a very luminous ILC, but are unlikely to be observable for large F . We have not computed the cross-sections, and suspect they are quite small since they are effectively loop-level.

An interesting possibility is that \tilde{B} couples to the electron, muon, or quarks directly and

can be produced in the s -channel. Then we have interesting processes such as:

$$e^+e^- \text{ or } q\bar{q} \text{ or } \mu^+\mu^- \rightarrow (\gamma^*, Z^*, \tilde{B}^*) \rightarrow \tilde{Z} + Z + (0, 1, 2)h^0; \tilde{Z} + \gamma + (0, 1, 2)h^0; WW + (0, 1, 2)h^0, \text{etc.} \quad (23)$$

Processes such as $e^+e^- \rightarrow \tilde{B}^* \rightarrow ZZ$ could interfere against normal EW physics, such as $e^+e^- \rightarrow ZZ$ producing an interference term that scales as N_c/F^2 with a chance at observability in detailed angular correlation studies at an ILC, CLIC or muon collider. Polarization may be a useful attribute to study in such processes.

Finally, as we have emphasized, even if \tilde{B} has no direct coupling to light fermions, it will necessarily decay through the T -parity violating processes:

$$\tilde{B} \rightarrow \tilde{Z} + Z + (0, 1, 2)h^0; \tilde{Z} + \gamma + (0, 1, 2)h^0; WW + (0, 1, 2)h^0, \text{ etc.} \quad (24)$$

As an explicit application, we compute the partial width:

$$\Gamma(\tilde{B} \rightarrow ZZ) \approx \frac{1}{2\pi} \left(\frac{\tilde{g}^3 N_c}{144\pi^2} \right)^2 \frac{m_Z^2}{m_{\tilde{B}}}, \quad (25)$$

to leading order in $\sin\theta_W$. Here we have used the relation $m_{\tilde{B}} = \tilde{g}F$ to simplify the result.

We have ignored the π and axion- η relic pNGB's which remain in the physical spectrum. These will also have associated anomalous interactions, such as $\pi^0 \rightarrow VV$, $a' \rightarrow VV$, where V is a vector boson. In extensions of this minimal model, we could gauge, *e.g.*, two copies of $SU(2)$, such that π would be eaten by a heavy \tilde{W} gauge boson. Note that the pNGB's correspond to axial generators and are always odd under T parity, while B and W correspond to vector generators and are therefore even. That \tilde{B} and \tilde{W} have a definite transformation (odd) under T parity relies on a further assumption of equality between coupling constants, $g_L = g_R$.

POPULAR LITTLE HIGGS MODELS

We presently survey a number of models in the literature for which an apparent T -parity can be defined. Most of these models are incomplete, and additional anomaly-cancelling structure is necessary for consistency.

The gauge structure of such a model must be sufficiently rich to leave an unbroken $SU(2) \times U(1)$ symmetry to be identified with the EW interactions. A model that incorporates one-loop cancellation of radiative corrections to the Higgs mass [3] also necessarily gauges broken

$U(1)$ (and $SU(2)$) generators. The same phenomenon occurs in more general composite models designed to break EW symmetry by vacuum misalignment [2]. Since the strong dynamics prefers a vacuum orientation that preserves EW symmetry, the misalignment can be achieved only if a broken $U(1)$ generator is gauged with sufficient strength. These broken $U(1)$ symmetries are, in general, anomalous, and entail the existence of an additional sector for anomaly cancellation.

The symmetry-breaking pattern $SU(n_f) \times SU(n_f) \rightarrow SU(n_f)$ is expected for a condensate of $2n_f$ Weyl fermions, with n_f of these in the fundamental representation of a strong color group $SU(N_c)$, and n_f in the anti-fundamental. $n_f = 3$ is the smallest value for which the “Little-Higgs cancellation” of one-loop mass corrections can be implemented. The existence of the WZW term can be traced to the nontrivial homotopy group for $SU(n) \times SU(n)/SU(n) = SU(n)$: $\pi_5(SU(n)) = \mathbf{Z}$ (for $n \geq 3$). The conclusions drawn from the $SU(3) \times SU(3) \rightarrow SU(3)$ model apply more generally. We mention here some specific examples.

$SU(5)/SO(5)$

The symmetry breaking pattern $SU(n_f) \rightarrow SO(n_f)$ is expected for a condensate of n_f Weyl fermions in a real representation of a strong color group. $n_f = 5$ is the smallest value for which the Little-Higgs cancellation can be implemented. We note that $\pi_5(SU(N)/O(N)) = \mathbf{Z}$ (for $n \geq 3$). This theory is described by a chiral field Σ transforming as

$$\Sigma \rightarrow e^{i\epsilon} \Sigma e^{-iR(\epsilon)}, \quad (26)$$

where $R(t^a) = \pm t^a$ for the unbroken and broken generators, respectively. [28] The WZW term for this model was derived in [5] and is similar to (9). Again, the action is unacceptable in isolation and requires an anomaly cancelling sector. This will again cancel the pure $\tilde{B}W\partial W$ and $\tilde{B}B\partial B$ terms, leaving allowed $\tilde{B}H^\dagger HW\partial W$ and $\tilde{B}H^\dagger HB\partial B$ terms [5]. The same general arguments apply here as in the previous case: T -parity is violated, the model is incomplete without an anomaly cancelling sector, and \tilde{B} is unstable.

Note that $\pi_3(SU(5)/SO(5)) = \mathbf{Z}_2$, and the theory must contain a skyrmion. The skyrmion reflects “baryons” at the scale Λ , and dictates the need for the WZW term to generate the corresponding Goldstone-Wilczek current.

$SU(6)/Sp(6)$

The symmetry breaking pattern $SU(n_f) \rightarrow Sp(n_f)$, for n_f even, is expected for a condensate of fermions in a pseudo-real representation of a strong color group. $n_f = 6$ is the smallest value for which the Little-Higgs cancellation can be implemented [19]. Again we note that $\pi_5(SU(n)/Sp(n)) = \mathbf{Z}$ (for $n \geq 4$). The low energy theory is a two-Higgs doublet model. We do not pursue further details here, but note again that a T -parity can be defined to act such that unbroken and broken generators are even and odd under T -parity. The WZW term is odd under T -parity, and mediates transitions between T -even and T -odd states. [29]

It is interesting that, while the WZW term exists here, there is evidently no skyrmion, since $\pi_3(SU(6)/Sp(6)) = 0$. A similar situation arises in the Kaplan-Schmaltz models [5] where we cannot construct a Goldstone-Wilczek current.

$[SU(3)/SU(2)]^2$

It is possible to search for an alternate definition of T -parity. For example in Eq.(18), we could consider $U_1 \leftrightarrow U_2$. This is a symmetry of the chiral Lagrangian when the WZW term is omitted, but is broken once it is included. This fact simply reflects the underlying chirality of the fermions: A_L is coupled to a left-handed fermion in U_1 , but to a right-handed fermion in U_2 . The same conclusions hold in various limiting cases. For example, strongly gauging a full $SU(3)_R$ in (18) results in the $[SU(3)/SU(2)]^2$ Kaplan-Schmaltz theory [16]; explicit T -parity violating interactions for this case were derived in Ref. [5].

$[SU(3) \times SU(3)/SU(3)]^4$

Consider the extension of (18) to a situation with *four* distinct condensates,

$$\begin{aligned} U_1 &\sim \Psi_{1L} \bar{\Psi}_{1R} \rightarrow e^{i\epsilon_L} U_1 e^{-i\epsilon_R}, \\ U_2 &\sim \Psi_{2R} \bar{\Psi}_{2L} \rightarrow e^{i\epsilon_L} U_2 e^{-i\epsilon_R}, \\ U_3 &\sim \Psi_{3L} \bar{\Psi}_{3R} \rightarrow e^{i\epsilon_L} U_3 e^{-i\epsilon_R}, \\ U_4 &\sim \Psi_{4R} \bar{\Psi}_{4L} \rightarrow e^{i\epsilon_L} U_4 e^{-i\epsilon_R}. \end{aligned} \tag{27}$$

For example, the model of [18] is of this form, with ϵ_L generating a full $SU(3)$, and ϵ_R generating $SU(2) \times U(1)$. The implementation of T -parity in a variant of this model via $L \leftrightarrow R$ as proposed in [10] suffers the same fate as the models already considered. However, the interchange symmetries $U_2 \leftrightarrow U_4$ (or $U_1 \leftrightarrow U_3$) are potentially valid symmetries of the full action [30]. We remark that this model has skyrmion solutions as well.

DISCUSSION

Composite Higgs models may provide a plausible explanation of EW symmetry breaking. Anomaly considerations have a crucial impact on the physics. Far from being a nuisance, the anomaly interactions of composite or Little Higgs theories provide a pathway to underlying UV physics that is accessible at the “low” energies that will be probed in the next generation of colliders. Perhaps the most dramatic effects are seen in the decays of new heavy particles that would otherwise be stable. This phenomenon is exactly analogous to the decay of $\pi^0 \rightarrow \gamma\gamma$ which specifies $N_c = 3$ in the QCD chiral Lagrangian.

When the NGB of interest is the kaon, considerations of conserved $SU(2)_W$ isospin symmetry (relatedly, strangeness) imply that the anomaly interactions are more difficult to observe. Single H interactions are forbidden by isospin, while two- H interactions with only massless gauge bosons are forbidden by parity. Thus nontrivial interactions start with two Higgs fields, plus either additional NGB’s, or massive gauge fields. Nonetheless, such interactions can have important implications, *e.g.*, giving rise to the T -parity violating decay mode studied in (25).

The mechanism of collective symmetry breaking encoded in many such models leads to a doubling of certain degrees of freedom, and it is natural to consider whether a new “ T ” parity can be defined as an exact symmetry. We described the general mechanism by which T -parity is violated in a Little Higgs model. It may be noted that although the discussion has been framed in terms of an underlying fermion UV completion, similar considerations hold for any UV completion in which the WZW term does not vanish.

In what situation can an exact T -parity be defined? One could consider flavor symmetry-breaking patterns, such $SO(n) \times SO(n)/SO(n)$, for which the flavor symmetry representations are real, and no anomalies occur. However, we do not know of any reasonable UV theories where such a symmetry-breaking pattern arises—certainly not in a theory of

strongly-interacting fermions. Even in familiar cases such as $SU(n) \times SU(n)/SU(n)$, it is a logical possibility that the integer coefficient of the WZW term is zero, but we do not know what UV completion could possibly lead to this. Reconstruction suggests that $N_c = 0$ would be a property of the equivalent $D = 5$ Yang-Mills theory, *i.e.*, the Chern-Simons term is absent for that theory. Even in the absence of fermions, such a theory contains instantonic solitons [12] (*i.e.*, the Euclidean $D = 4$ instanton is a world-line) and these match onto the Skyrmion under compactification. Their currents and various static properties, like spin and statistics, are controlled by the Chern-Simons term, which matches onto the WZW term. $D = 5$ Yang-Mills does require a UV completion, but to have $N = 0$ would require a suppression of all topological aspects of that theory by the completion theory.

Alternatively, T -parity can be defined as an exchange symmetry between sectors with identical chiral fermion content, *cf.* the discussion after Eq.(27). Whether this symmetry can be maintained after adding standard model fermions, and terms generating a Higgs potential, remains to be explored.

It is important to work out the phenomenological implications of broken T -parity; in particular, missing energy signals at colliders arising from a stable \tilde{B} should be reconsidered [20]. Certainly predictions of a dark matter candidate based on naive T -parity need to be revised [21, 22]. Independently of whether an approximate or exact T -parity can be found, it is interesting to look for observable effects of the spectator lepton sector that is necessary to cancel the anomalies of a general Little Higgs model. Understanding the spectator sector is an important problem. The interplay of this sector with the strong interactions could in itself provide a new mechanism for causing vacuum misalignment, and EW symmetry breaking.

We emphasize that these considerations will generally apply to any models of extra dimensions where Chern-Simons terms appear (*cf.* the second paper in the sequence of Ref. [15]). For example, chiral delocalization appears in Randall-Sundrum schemes such as the “composite top” models [23]. The CS term must be included and will involve gravitational as well as gauge interactions. We should also revisit the question of when the lightest KK-mode in higher dimensional models is stable in the presence of a CS term [24].

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- [25] For example, the instanton becomes a world-line (particle) in $D = 5$ and has two associated conserved topological currents. These currents can only be generated if Chern-Simons terms are appended to the theory [12].
- [26] In a more precise language, Chern-Simons operators arise only in odd D , as in $\epsilon^{ABCDE} A_A d_B A_C d_D A_E$. Here we use the pejorative meaning for $D = 4$ operators such as $\epsilon_{\mu\nu\rho\sigma} A^\mu B^\nu d^\rho B^\sigma$ or $\epsilon_{\mu\nu\rho\sigma} \text{Tr}(A^\mu B^\nu B^\rho B^\sigma)$.
- [27] Note that we can also include a matrix Ω with $\Omega^2 = 1$ into the definition of T -parity: $\tilde{\pi} \rightarrow -\Omega\tilde{\pi}\Omega$, *etc.* For example, $\Omega = \text{diag}(1, 1, -1)$ will allow the K to be defined as scalar, whereas the π and η are pseudoscalar. As in the $SU(3)$ QCD chiral Lagrangian, this has no physical consequences until isospin-violating interactions are added.
- [28] Alternatively, the low-energy theory can be arrived at by embedding inside of $SU(5) \times SU(5)/SU(5)$ with the extraneous NGB's removed by a strongly-coupled gauge field.
- [29] Note that in contrast to the $U(1)$ charges advocated in Ref. [19], it is possible to choose generators that do not induce additional $U(1)$ /color anomalies — *e.g.*, in the coordinates of Ref. [19], $Y_1 \propto (1, 1, 1, 1, 1, -5)$, and $Y_2 \propto (1, 1, -5, 1, 1, 1)$.
- [30] The mechanism of [18] used to stabilize a flat direction in the Higgs potential breaks the $U_2 \leftrightarrow U_4$ symmetry.